Conservative Regridding for Spherical Surfaces

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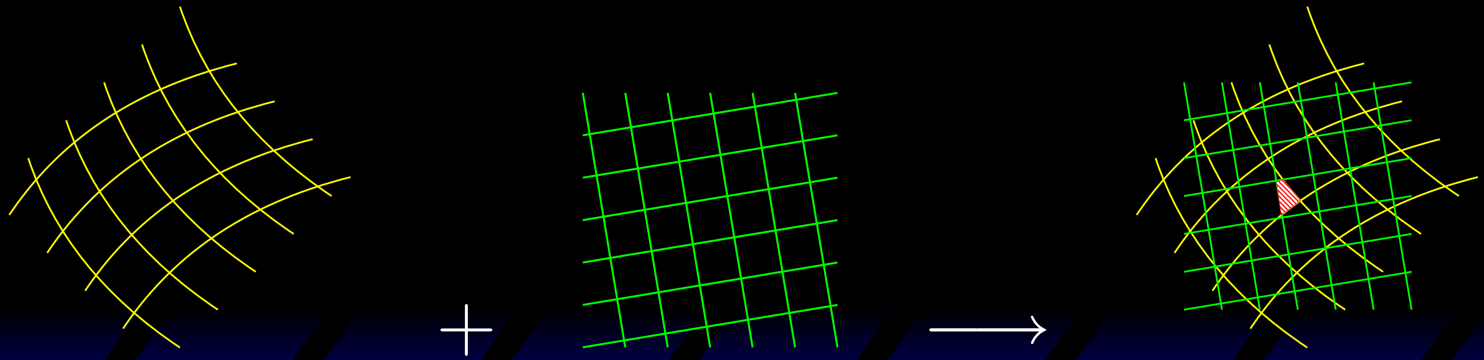
Motivation

Re-mapping or re-gridding is an important problem:

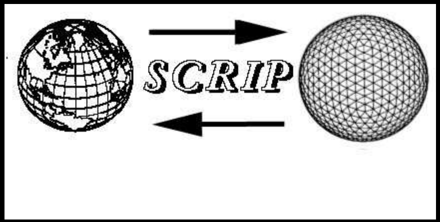
- get your input data onto the model grid
- get the model results into a form usable by others
- sometimes, a simple/crude interpolation is sufficient
- frequently, a **conservative** scheme is required/desired
  - ideally, we'd like to exactly (to machine precision) preserve: mass, thermal energy, radiative fluxes, etc.
  - one may also want to preserve (or minimize/maximize) some norm of stress, energy, momentum, etc.



Background

Current state-of-the-art is SCRIP [P. Jones, LANL]

http://climate.lanl.gov/Software/SCRIP/



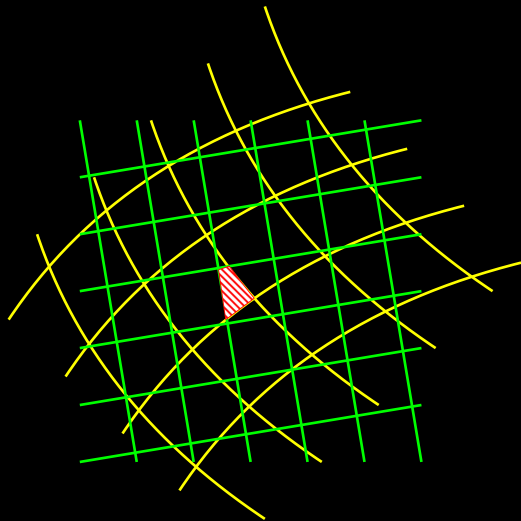
- uses Gauss Divergence theorem to reduce area integrals to simpler and more efficient line integrals around the boundaries of overlapping regions
- has first- and second-order accurate methods with regard to non-constant functions of two spatial dimensions

Background

SCRIP needs improvement or augmentation in the following areas:

- Numerical integration is only accurate to a few significant figures.
- SCRIP does not faithfully represent cell areas for non-lat-lon grids (all computations are done on an assumed/underlying lat-lon grid).
- How does one conservatively regrid vector quantities? What do you conserve?
  - integrated mass fluxes
  - energy, momentum, etc.
  - fix one norm while maximizing or minimizing another

A New Method

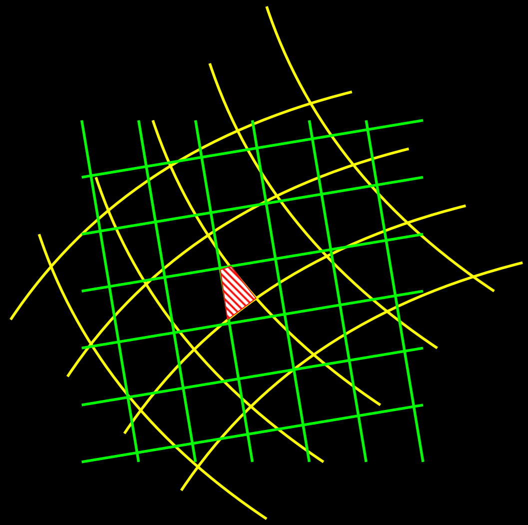


Treat each grid cell as a *feasible region* defined by constraints of the form:

$$\begin{aligned} \mathbf{d}_i \cdot \mathbf{p} &\geq \alpha_i \quad \forall i = 1, \cdots, n \\ \|\mathbf{p} - \mathbf{c}\|_2 &= r \end{aligned}$$

where $\mathbf{p}$ is any point, $\mathbf{d}_i$ is a unit direction vector, α_i is a scalar constant, $\mathbf{c}$ is the center of the sphere, and r is the sphere radius.

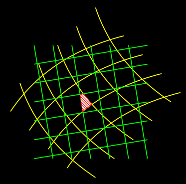
A New Method



The *intersection* of any two grid cells is the *union* of their constraints.

- no feasible region $\longrightarrow$ no overlap
- If a feasible region does exist, then one can precisely (to machine precision!) determine:
  - intersection points (the verticies)
  - shared arcs (the active contraints)
  - the area of the feasible region

A New Method



With exact areas, lengths, etc., we can construct conservative regridding schemes for:

- scalars
- vectors

In many cases, the schemes result in *local* problems – the scope is limited to a region defined by the coarser of the two grids.

A New Method

Goals are:

- inclusion (in an on-line sense) within MITgcm
- inclusion of the “computational kernel” as a regridding option (alongside SCRIP, etc.) within ESMF